ENERGY LOSSES IN A GAS-PARTICLE SYSTEM DUE TO THE IRREVERSIBILITY OF INTERPHASE HEAT TRANSFER

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The energy losses in a dust-gas suspension due to heat transfer between the gas and solid phases are found by the methods of the thermodynamics of irreversible processes. An expression is obtained for the energy losses in the approximation of the thermal relaxation time of the particle.

In any thermodynamic process the energy losses per unit time are given [1] by

$$\dot{E} = T_0 \dot{S}. \tag{1}$$

The rate of entropy production in a system of volume V can be represented in the form [2]

$$\dot{S} = \frac{dS}{dt} = \int_V \sigma\, dV. \tag{2}$$

Let us consider a system consisting of a gas in which solid particles are suspended. Regarding the particles as point sources of entropy distributed through the system with density [3]

$$n(\mathbf{r}) = \sum_j^N \delta(\mathbf{r} - \mathbf{r}_j),$$

and assuming that collisions between particles do not affect the net entropy production and that the superposition principle is satisfied, we write the expression for the entropy production (per unit volume of suspension)

$$\sigma = \sum_j^N \dot{S}_j\, \delta(\mathbf{r} - \mathbf{r}_j). \tag{3}$$

The rate of increase of entropy $\dot{S}_j$ due to heat transfer between the gas and the j-th particle [2] is equal to

$$\dot{S}_j = \int_{V'} \sigma\, dV' = -\int_{V'} \left[\frac{(\mathbf{q}\cdot\vec{\nabla} T)}{T^2}\right]_j dV'. \tag{4}$$

Here, by virtue of the superposition principle, the integration is carried out over the entire volume ($V' \equiv V$) in the coordinate system tied to the j-th particle. Substituting (4) into (3), and then (3) into (2) and (1), we obtain

$$\dot{E} = -T_0 \sum_{j=1}^N \int_V \delta(\mathbf{r} - \mathbf{r}_j) \int_{V'} \left[\frac{(\mathbf{q}\cdot\vec{\nabla} T)}{T^2}\right]_j dV' dV. \tag{5}$$

Using (5), we can calculate $\dot{E}$ for any given particle distribution function $n(\mathbf{r})$.

We consider a simple disperse system in which the solid phase, in the form of spherical particles of identical size ($R_j \equiv R$), is uniformly distributed over the volume of the system. We assume that Bi → 0 and that at any time the system consists of two equilibrium subsystems: a gas at temperature T_g and particles at temperature T_s. The gas-particle temperature distribution has the form (Fig. 1)

$$T = \begin{cases} T_s & \rho \leqslant R, \\ T_g & \rho > R. \end{cases}$$

For this distribution, given spherical symmetry, the temperature gradient has the form

$$\frac{\partial T}{\partial \rho} = A(t)\, \delta(\rho - R). \tag{6}$$

Integrating (6) with respect to ρ from 0 to $R + \varepsilon$ ($\varepsilon > 0$) by virtue of the properties of the delta function [4] we obtain for the normalization factor A(t)

$$A = \Delta T(t) = T_g - T_s.$$

Then

$$\frac{\partial T}{\partial \rho} = \Delta T\, \delta(\rho - R). \tag{7}$$

Using (7), we can easily evaluate the inner integral in (5) in spherical coordinates ($dV' = 4\pi\rho^2 d\rho$):

$$\int_{V'} \left[\frac{(\mathbf{q}\,\vec{\nabla} T)}{T^2}\right]_j dV' =$$

$$= -\frac{q(R)}{T_s^2}\frac{\Delta T}{R_j} 4\pi R_j^3 = -\frac{\alpha}{R_j}\left(\frac{\Delta T}{T_s}\right)^2 4\pi R_j^3, \tag{8}$$

where $q(R) = \alpha\Delta T$ according to Newton's law.

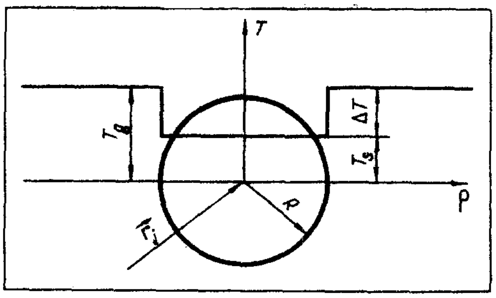

Fig. 1. Temperature distribution in the gas and in the particle.

For a system of N identical spherical particles

$$N = \mu \frac{\rho_g}{\rho_s} \frac{3}{4\pi R^3} V_g, \tag{9}$$

and, using (8) and (9), we obtain expression (5) in the form

$$\dot{E} = 3T_0 \frac{\alpha}{R}\left(\frac{\Delta T}{T_s}\right)^2 \mu \frac{\rho_g}{\rho_s} V_g. \tag{10}$$

Integrating (10) with respect to time and setting $\rho_g V_g \equiv \equiv 1$, we find the energy losses referred to unit mass of gas in an arbitrary thermodynamic process:

$$\Delta E = 3\mu T_0 \frac{\alpha}{R\rho_s} \int\limits_0^\tau \left(\frac{\Delta T}{T_s}\right)^2 dt. \tag{11}$$

The quantity ΔE does not depend on the direction of the heat flow.

Let us evaluate the integral in expression (11) for processes whose duration exceeds the relaxation time of the subsystem with the greater inertia. At $\mu > 1$ $\tau_g < \tau_s$. Hence, we assume that the gas changes temperature instantaneously, while the particles relax to the end state of the gas (Fig. 2):

$$T_g = \begin{cases} T_{g1} = T_{s1}, & t < 0, \\ T_{g1} \pm \Delta T_g, & t \geqslant 0. \end{cases} \tag{12}$$

The quantity $\Delta T = T_g - T_s$ can be found from the particle heating (cooling) equation as $\mathrm{Bi} \to 0$

$$\frac{\rho_s c_s R}{3\alpha} \frac{dT_s}{dt} = (T_g - T_s) = \Delta T.$$

Using (12), we obtain the solution of this equation for ΔT in the form

$$|\Delta T| = |\Delta T_g| \exp\left\{-\frac{t}{\tau_s}\right\},$$

where $\tau_s = \rho_s c_s R / 3\alpha$.

The integral in (11) is equal to

$$\int\limits_0^\tau \left(\frac{\Delta T}{T_s}\right)^2 dt \approx \int\limits_0^\infty \left(\frac{\Delta T_g}{T_{g1}}\right)^2 \exp\left\{-\frac{2t}{\tau_s}\right\} dt =$$

$$= \left(\frac{\Delta T_g}{T_{g1}}\right)^2 \frac{\tau_s}{2} = \theta^2 \frac{\tau_s}{2};$$

θ is the relative change of gas temperature during the process.

In integrating it was taken into account that $\Delta T \ll \ll T_{g1}$ and, moreover, $\tau \gg \tau_s$, which permits $\tau \to \infty$. Hence, from (11) we obtain

$$\Delta E = \frac{1}{2} \mu T_0 c_s \theta^2.$$

The value of θ is determined by the nature of the thermodynamic process and can be found if the polytropic index n, whose value depends on the concentration μ, the heat-transfer coefficient α, the thermophysical parameters of the gas and the particles, and the rate of the process [5], is known. The energy losses, calculated from (13), for the externally adiabatic compression of an air-graphite suspension are presented in Fig. 3. The value of θ was calculated from the equation of the polytropic process for two values of the polytropic index, $n_1 = 1.072$ and $n_2 = 1.067$, obtained experimentally [5] at concentrations $\mu_1 = 4.5$ and $\mu_2 = 10.9$, respectively. The temperature and specific heat were taken equal to $T_0 = 300°$ K, $c_s = 1$ kJ/kg • deg.

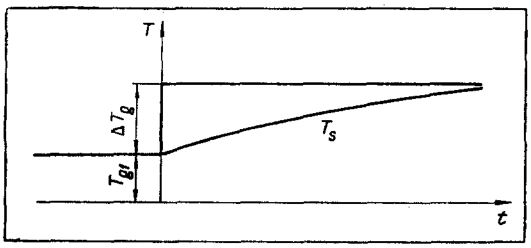

Fig. 2. Variation of gas and particle temperatures with time.

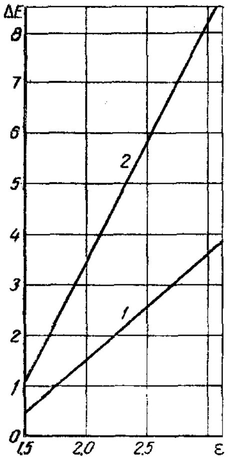

Fig. 3. Energy losses (kJ/kg) in a suspension ($\varepsilon = p_2/p_1$ is the compression ratio): 1) $\mu = 4.5$; 2) $\mu = 10.9$.

NOTATION

$\dot{E}$ are the energy losses per unit time; T_0 is the temperature of surrounding medium; $\dot{S}$ is the rate of increase of entropy; V is the volume of the thermodynamic system; σ is the entropy production; n(r) is the particle distribution function; **q** is the heat flux density; $\vec{\nabla} T$ is the temperature gradient; R is the particle radius; ρ is the variable radial coordinate in system tied to particle; T_g is the gas temperature; T_s is the particle temperature; α is the heat-transfer coefficient; N is the number of particles; μ is the mass concentration; ρ_s is the density of the particle material; ρ_g is the gas density; V_g is the volume occupied by the gas; ΔT_g is the change of the gas temperature during the process; T_{g1} is the initial gas temperature; Bi is the Biot number; t is the time; $\delta(\mathbf{r} - \mathbf{r}_j)$, $\delta(\rho - R)$ is the Dirac delta function; $\mathbf{r}_j$ is the particle radius vector; τ_g is the gas relaxation time; τ_s is the particle relaxation time; τ is the process time.

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